The correlation method

...is really just the application of a textbook example for quantum mechanics

Basic concepts

- **Separation of a vector wave function into space dependent and time dependent factors**

  A standard method taught in introductory courses of both classical electrodynamics and quantum mechanics is the separation of a function that depends on several coordinates into a product of functions, each of which depends only on a subset of the coordinates.
  It is particularly common to separate stationary wave functions, i.e., functions whose observable characteristics remain essentially the same over the period of observation, in the product of a function depending only on the spatial coordinates and a time dependent function:
  http : // pauli.uni – muenster.de / menu / Lehre / quant – skript / node11.html
  In the case of complex wave functions, the complex nature of the wave function can be entirely absorbed into the time dependent function.
  The standard examples in quantum mechanics are stationary states, e.g. in a box potential
  http : // pauli.uni – muenster.de / menu / Lehre / quant – skript / node13.html,
  a harmonic oscillator http : // pauli.uni – muenster.de / menu / Lehre / quant – skript / node31.html,
  or a hydrogen atom http : // pauli.uni – muenster.de / menu / Lehre / quant – skript / node60.html;
  the standard example in classical electrodynamics is the cavity resonator.
  It is therefore obvious to consider an electromagnetic wave as a product of a time dependent function and a (real vector) function dependent on space.
  Furthermore, it is obvious to consider the space dependent component function of an essentially time invariant function at a finite set of locations that do not change substantially within the time of observation to be essentially constant.

- **Decomposition of a vector wave function into coordinates**

  The basic formalism of a vector valued function described as a plurality of scalar valued function is taught in introductory university courses of physics, mathematics, and engineering, at least in Germany.

- **Coordinate transformation in space**

  Coordinate transformations in space, in particular rotations, are part of the basic curriculum in university courses of the subjects mentioned above
  http : // pauli.uni – muenster.de / menu / Lehre / quant – skript / node43.html.
  This includes rotations in more than three dimensions, e.g., in special relativity
  http://pauli.uni-muenster.de/Lehre/Skripten/Eckelt/edynrel.pdf.

- **Eigenvalues and eigenfunctions**

  Eigenvalues as observables and eigenvectors as the wave functions corresponding to the observables are an elementary concept obvious to anybody knowledgeable in classical quantum mechanics:
  http : // pauli.uni – muenster.de / menu / Lehre / quant – skript / node35.html,
  in particular of the Heisenberg matrix mechanics variety developed in the 1920s.

- **The statistical operator formalism**

  A concept of particular importance for the method of detecting electromagnetic field vectors by correlation is the statistical operator, which is taught in basic courses of quantum mechanics or statistical physics, e.g., http://pauli.uni-muenster.de/menu/Lehre/quant-skript/node99.html.
  At the time the concept of the statistical operator is introduced, it is obvious to students following the course that for a pure state, i.e. a state with non-vanishing off diagonal coherence terms, a coordinate transformation can be found that transforms the statistical operator into diagonal form.

- **Cavity radiation as *the* standard example of statistical physics**

  The standard example to introduce the concept of quantum statistics is the electromagnetic radiation in a cavity. The mathematically simplest, and therefore most obvious for somebody skilled in the art, treatment is the decomposition into Cartesian coordinates http://saftsack.fs.uni-bayreuth.de/thermo/elek.html.
  As in the correlation method developed by me as an obvious application of the concept, any mode of the cavity resonator is described as coherent superposition of the projection of the wave function on the coordinate directions.

A textbook example of the basic concepts

We will make heavy use of the notation introduced by the electrical engineer and physicist P.A.M. Dirac. For definitions, consult a textbook on introductory quantum mechanics.

- **Statistical operator for a stationary (electro-)magnetic field**

  We begin with the definition of the statistical operator

$$\rho \equiv \sum_\alpha |\alpha\rangle \, p_\alpha \, \langle\alpha|$$

  of an ensemble of pure states, indexed by α. (see http://pauli.uni-muenster.de/menu/Lehre/quant-skript/node98.html for the textbook definition).

  The positive real coefficients p_α correspond to a probability of a certain state, or in case of many particle systems, the amplitude of a mode of the field.
  If the vectors are assumed to describe photons, we can represent the photon density by the proportional magnetic field, i.e.,

$$\rho(\vec{x}, t) \equiv \sum_\alpha \left| \frac{\vec{B}_\alpha(\vec{x},t)}{\left|\vec{B}_\alpha(\vec{x},t)\right|} \right\rangle \vec{B}_\alpha^{\,2}(\vec{x}, t) \left\langle \frac{\vec{B}_\alpha(\vec{x},t)}{\left|\vec{B}_\alpha(\vec{x},t)\right|} \right| = \sum_\alpha \left|\vec{B}_\alpha(\vec{x}, t)\right\rangle \left\langle \vec{B}_\alpha(\vec{x}, t)\right|$$

  Since, for our simple and obvious example, we don't want to be bothered by the dynamics of the electromagnetic field, we'll assume that the electromagnetic field is essentially stationary in the time interval we're observing it, i. e., we can factor the field into a space dependent and a time dependent component.

$$\left|\vec{B}_\alpha(\vec{x}, t)\right\rangle \equiv \left|\vec{B}_\alpha(\vec{x})\right\rangle \otimes |\beta_\alpha(t)\rangle$$

  If we're interested in classically observable electromagnetic fields, we can make $\left|\vec{B}_\alpha(\vec{x})\right\rangle$ a real vector valued function. For the time being, we'll allow $|\beta_\alpha(t)\rangle$ to be a complex scalar function.
  We'll also assume that we'll stay essentially in one place during one measurement, so we can drop the

index $\vec{x}$ in our further discussion.

To take care of the time dependent part of the wave function, we'll define

$$\forall\ \alpha:\ \langle\beta_\alpha(t)\ |\ \beta_\alpha(t)\rangle \equiv \sum_t \beta_\alpha{}^*(t)\,\beta_\alpha(t) \equiv \sqrt{N}$$

and get on with the discussion.

We can always do this sort of normalization, because we can move a real constant that depends neither on space nor time between the space and the time dependent factors.

Looking at the space dependent variable in just one point in space, we're left with a constant vector $\vec{B}_\alpha$ for each state α.
So, each state is represented by a constant field vector and a normalized time-dependent part.

As next step, we get rid of the time variable by taking the trace over the time variable:

$$\rho = \sum_\alpha \left(\left|\vec{B}_\alpha\right\rangle \left\langle\vec{B}_\alpha\right| \otimes \langle\beta_\alpha(t)\right| \left(\left|\beta_\alpha(t)\right\rangle \langle\beta_\alpha(t)\right|\right) \left|\beta_\alpha(t)\rangle\right) = N \sum_\alpha \left|\vec{B}_\alpha\right\rangle \left\langle\vec{B}_\alpha\right|$$

Now let's assume that we have a pure state, i.e., only one non-zero field vector $\vec{B}_\aleph$. We can find an orthonormal basis

$$\left|\vec{\epsilon}_i\right\rangle,\ \ i \in \{1,\ 2,\ 3\} \text{ with } \left\langle\vec{\epsilon}_j \,\middle|\, \vec{\epsilon}_i\right\rangle = \delta_{i\,j}$$

such that the first basis vector is parallel to the field vector:

$$\left\langle\vec{\epsilon}_i \,\middle|\, \vec{B}_\aleph\right\rangle = \left|\vec{B}_\aleph\right| \delta_{i\,j}$$

In this representation, the matrix elements of the statistical operator are

$$\left\langle\vec{\epsilon}_j \,\middle|\, \rho \,\middle|\, \vec{\epsilon}_i\right\rangle = N \left\langle\vec{\epsilon}_j \,\middle|\, \vec{B}_\aleph\right\rangle \left\langle\vec{B}_\aleph \,\middle|\, \vec{\epsilon}_i\right\rangle = N\, \vec{B}_\aleph{}^2\, \delta_{1\,i}\, \delta_{1\,j}$$

or, written as a matrix,

$$\overleftrightarrow{\rho} = \begin{pmatrix} N\, \vec{B}_\aleph{}^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

We can transform the basis into another orthonormal basis $\left|\vec{e}_i\right\rangle,\ \ i \in \{1,\ 2,\ 3\}$ with $\left\langle\vec{e}_j \,\middle|\, \vec{e}_i\right\rangle = \delta_{i\,j}$ by an orthogonal transformation, i.e., a rotation.

Since, by construction, $\vec{B}_\aleph \parallel \vec{\epsilon}_1$, the vector $\vec{B}_\aleph$ can be expressed in the $\left|\vec{e}_i\right\rangle$ basis as

$$\vec{B}_\aleph = \sqrt{\frac{N\, \vec{B}_\aleph{}^2}{N}} \begin{pmatrix} \left\langle\vec{e}_1 \,\middle|\, \vec{\epsilon}_1\right\rangle \\ \left\langle\vec{e}_2 \,\middle|\, \vec{\epsilon}_1\right\rangle \\ \left\langle\vec{e}_3 \,\middle|\, \vec{\epsilon}_1\right\rangle \end{pmatrix}$$

In this basis, the matrix elements of the statistical operator are

$$\langle \vec{e}_j \mid \rho \mid \vec{e}_i \rangle = N \langle \vec{e}_j \mid \vec{B}_\varkappa \rangle \langle \vec{B}_\varkappa \mid \vec{e}_i \rangle \equiv N\, B_j\, B_i$$

i.e., the elements of the positive definite, symmetric correlation matrix

$$\overleftrightarrow{\rho} = N \begin{pmatrix} B_1{}^2 & B_1\, B_2 & B_1\, B_3 \\ B_1\, B_2 & B_2{}^2 & B_2\, B_3 \\ B_1\, B_3 & B_2\, B_3 & B_3{}^2 \end{pmatrix}$$

A method that transforms this matrix into diagonal form will yield both the eigenvalue $\vec{B}_\varkappa^{\,2}$ and the eigenvector $\begin{pmatrix} \langle \vec{e}_1 \mid \vec{\epsilon}_1 \rangle \\ \langle \vec{e}_2 \mid \vec{\epsilon}_1 \rangle \\ \langle \vec{e}_3 \mid \vec{\epsilon}_1 \rangle \end{pmatrix}$ required to determine $\vec{B}_\varkappa$.

Methods to diagonalize a positive definite, symmetric matrix are included in college courses on linear algebra and/or numerical mathematics and therefore a well known part of the physics curriculum at German universities.

If several states (i.e. fields in several directions that are not zero all the time) are present simultaneously, the diagonalized statistical operator will have more than one non-zero diagonal matrix element.

However, if we are interested in the direction of the largest field magnitude, this direction obviously corresponds to the eigenvector to the largest eigenvalue.

The statistical operator is introduced in a context in which the concept of a Hilbert space has already been introduced. Therefore we can safely assume that the generalization of this example for more than 3 dimensions, by defining states by the essentially stationary magnetic field and/or projections on certain spatial directions thereof at several points in space, is an obvious generalization of the trivial example of a statistical operator detailed above.

A closer look at the time dependent factor

The normalization of the time dependent factor $|\beta(t)\rangle$ above was chosen such that a simple summation of spatial correlation matrices $\overleftrightarrow{\rho}$ at N different points in time (that need not be spaced regularly) adds up to N. This makes sense since a sum of measurements of a stationary magnetic field is essentially proportional to the number N of measurements.

The set of time dependent functions $|\beta(\mathrm{t})\rangle$ form a Hilbert space. For this Hilbert space, several orthonormal sets of basis vectors exist, e.g., the unit impulse functions $\{|\delta(t-t_0)\rangle\}_{t_0\in\mathbb{R}}$ and the Fourier decomposition into real waves $\{|\cos(2\pi f t)\rangle\}_{f\in\mathbb{R}} \bigcup \{|\sin(2\pi f t)\rangle\}_{f\in\mathbb{R}}$ or complex waves $\{|\exp(2\pi i f t)\rangle\}_{f\in\mathbb{R}}$.

The statistical operator whose representation in space has been discussed above can be restricted on a class of time dependencies by projection of the time dependent factor on a subspace of the Hilbert space of scalar time dependent functions, i.e. on a finite subset of basis vectors. An obvious example, familiar from the example of electromagnetic radiation in a cavity used to introduce the concept of quantum statistics, is the projection of discrete frequencies. As the simplest case, we shall project on one frequency only, i.e. one complex basis vector $|\exp(2\pi i f t)\rangle$ or two real basis vectors $\{|\cos(2\pi f t)\rangle, |\sin(2\pi f t)\rangle\}$.

The statistical operator projected on harmonic signals of one frequency f becomes

$$\rho(f) = \sum_\alpha \left(\left|\vec{B}_\alpha\right\rangle \left\langle \vec{B}_\alpha \right| \otimes \langle \beta_\alpha(t) | \left(|\exp(2\pi i f t)\rangle \langle \exp(2\pi i f t)| \right) |\beta_\alpha(t)\rangle \right) = \tilde{\beta}_\alpha^*(f)\, \tilde{\beta}_\alpha(f) \sum_\alpha \left|\vec{B}_\alpha\right\rangle \left\langle \vec{B}_\alpha \right|$$

where $\tilde{\beta}_\alpha(f) \equiv \langle \exp(2\pi i f t) \mid \beta_\alpha(t)\rangle$is the complex Fourier coefficient of $\beta_\alpha(t)$ for frequency f and * denotes the complex conjugate.

Following the argumentation above, the magnitude and orientation in space of largest *periodic* signal at frequency f can be found by diagonalizing a Hermitian matrix

$$\overleftrightarrow{\rho} = N \begin{pmatrix} B_1{}^* B_1 & B_1{}^* B_2 & B_1{}^* B_3 \\ B_2{}^* B_1 & B_2{}^* B_2 & B_2{}^* B_3 \\ B_3{}^* B_1 & B_3{}^* B_2 & B_3{}^* B_3 \end{pmatrix}$$

with real eigenvalues and complex eigenvectors. The phase of the eigenvector components denote the phase of the periodic function $e^{2\pi i f t}$.

- **Canon cancricans**

We can also apply the concept above in reverse and diagonalize the time dependent part of the projection of the statistical operator on a finite set of spatial components at a finite set of locations and one particular frequency (this time we'll represent it in a real basis).

$$\rho = \sum_i \left(\langle \vec{e}_i | \otimes \begin{pmatrix} \langle \cos(2\pi f t)| \\ \langle \sin(2\pi f t)| \end{pmatrix} \right) \left|\vec{B}_\varkappa(t)\right\rangle \left\langle \vec{B}_\varkappa(t)\right| \left(|\vec{e}_i\rangle \otimes \left(|\cos(2\pi f t)\rangle \quad |\sin(2\pi f t)\rangle \right) \right) =$$
$$\sum_i \begin{pmatrix} \langle \cos(2\pi f t) \mid B_i(t)\rangle \langle B_i(t) \mid \cos(2\pi f t)\rangle & \langle \cos(2\pi f t) \mid B_i(t)\rangle \langle B_i(t) \mid \sin(2\pi f t)\rangle \\ \langle \sin(2\pi f t) \mid B_i(t)\rangle \langle B_i(t) \mid \cos(2\pi f t)\rangle & \langle \sin(2\pi f t) \mid B_i(t)\rangle \langle B_i(t) \mid \sin(2\pi f t)\rangle \end{pmatrix} \equiv$$
$$\begin{pmatrix} \vec{B}_I \cdot \vec{B}_I & \vec{B}_I \cdot \vec{B}_Q \\ \vec{B}_Q \cdot \vec{B}_I & \vec{B}_Q \cdot \vec{B}_Q \end{pmatrix}$$

We can also represent this projection of the statistical operator on waves with frequency f on another basis.

Let's take, for example, use a representation that distinguishes between time-invariant, and time-variant components of the statistical operator.

For starters, let's observe that the time dependence of all non-vanishing components of the projection of the statistical operator must have a time dependence that results from the products $\langle \cos(2\pi f t) \mid \cos(2\pi f t)\rangle = \frac{1}{2} + \frac{\cos(4\pi f t)}{2}$, $\langle \cos(2\pi f t) \mid \sin(2\pi f t)\rangle = \frac{\sin(4\pi f t)}{2}$, and $\langle \sin(2\pi f t) \mid \sin(2\pi f t)\rangle = \frac{1}{2} - \frac{\cos(4\pi f t)}{2}$.

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Last[Solve[{(d + c)/2 == BI BI, s/2 == BI BQ, (d - c)/2 == BQ BQ}, {c, d, s}]]
Simplify[c^2 + s^2 /. %]
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{c → BI^2 - BQ^2, d → BI^2 + BQ^2, s → 2 BI BQ}
(BI^2 + BQ^2)^2
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Obviousness

The correlation method for detecting the magnitude and orientation of electromagnetic vector fields of the form $\vec{B}(\vec{x},\ t) \equiv \vec{\mathrm{B}}(\vec{x}) \cdot \beta(t)$, at a location $\vec{x}$ with essentially arbitrary temporal behavior $\beta(t)$ is a straightforward application of the standard syllabus of theoretical physics taught at German universities, and thus obvious to anybody skilled in this art (i.e., knowledgeable of German college level physics).

- **Competent jurisdiction**

The method may not appear obvious to citizens of a country who aren't aware of quite a few obvious facts, for example, that confirming in office a government, the members of which should not be hanged for crimes against peace, war crimes, and crimes against humanity, if only for the reason that the death penalty is an unacceptable form of punishment for any civilized country, constitutes a serious impediment to conducting business in a spirit of mutual trust and cooperation, and thus generates unnecessary costs and other obstructions of business that would have been entirely avoidable otherwise.
It should also be noted that an electorate that confirms in office an administration that considers a war of aggression an acceptable means of policy proves clearly that it is either incapable of recognizing or simply refuses to recognize certain most elementary facts of nature and civilization which are prerequisite to understanding the above argumentation. As a necessary consequence, any jurisdiction that consists of or derives its authority from said electorate must not be considered nor made appear to be competent to judge the novelty or obviousness of the idea presented herein.

Nonetheless, the argument of obviousness (according to US judicial criteria) holds for citizens of civilized countries skilled in the art (but not of extraordinary ability, as determined by the criteria of US immigration jurisdiction), as I have detailed in this document.